Technical Notes

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Free Vibration of Transversely Isotropic Circular Plates

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DOI: 10.2514/1.18093

Introduction

CIRCULAR and annular plates have been widely used in various engineering applications and the associated dynamic study has always been of research interest [1–5]. Ding and Xu [6] proposed a general solution method to study the nonaxisymmetric free vibration of transversely isotropic circular plates. They found that exact three-dimensional solutions could be found for the following two kinds of boundary conditions at the boundary $r = a$:

$$\text{Boundary condition (i): } w = 0, \qquad u_\theta = 0, \qquad 2c_{66}u_r + a\sigma_r = 0 \tag{1a}$$

$$\text{Boundary condition (ii): } u_r = 0, \qquad \tau_{zr} = 0, \qquad 2c_{66}u_\theta + a\tau_{r\theta} = 0 \tag{1b}$$

where u_r, u_θ, and w are the displacement components in r-, θ-, and z-directions, respectively, in the cylindrical coordinate system (r, θ, z), σ_i and τ_{ij} are the normal and shear stresses, c_{ij} are the material constants, and we have $c_{66} = (c_{11} - c_{12})/2$ for transversely isotropic materials. On the other hand, the other available so-called exact three-dimensional analyses [2] did not deal with the above boundary conditions, and actually involved obvious mistakes, as discussed by Ding and Xu [7].

In this note, we try to get a deeper insight into the problem, as a necessary supplement to the paper of Ding and Xu [6]. Apart from the out-of-plane vibration considered in the paper of Ding and Xu, the in-plane vibration is simultaneously analyzed here. Such a treatment has not been performed yet, because the out-of-plane vibration was usually assumed to be not present when the in-plane vibration was studied and vice versa. It will be shown later, however, that the separation of in-plane vibration and out-of-plane vibration is a natural result due to the free surface conditions. It is further found that the symmetric mode vibration and the antisymmetric mode vibration, with respect to the middle plane of the plate, can be separated in the case of the out-of-plane vibration. A similar result was once obtained for simply supported rectangular plate [8,9]; however, no work on circular plate has been reported in literature. This is mostly due to the fact that it is very difficult to obtain exact three-dimensional solution of a circular plate, for which the boundary conditions, as shown in Eq. (1), are not so simple and straightforward as those for a rectangular plate.

It is noted that vibration mode shapes provide important information for design of electric devices as well as engineering structures [10–14].

Received 8 June 2005; revision received 20 December 2005; accepted for publication 2 January 2006.

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General Solution and Free Vibration Analysis

Consider a transversely isotropic circular plate with radius a, thickness h, and the plane of isotropy parallel to the middle plane $z = 0$. The geometry and coordinates are shown in Fig. 1. In cylindrical coordinates (r, θ, z) with z-axis perpendicular to the plane of isotropy, the general solution of equations of motion is [6]

$$u_r = -\frac{1}{r}\frac{\partial\psi}{\partial\theta} - \frac{\partial^2 F}{\partial r\partial z}, \qquad u_\theta = \frac{\partial\psi}{\partial r} - \frac{1}{r}\frac{\partial^2 F}{\partial\theta\partial z}$$
$$w = \left(a_0\Lambda + b_0\frac{\partial^2}{\partial z^2} - \frac{b_0}{c_{44}}\rho\frac{\partial^2}{\partial t^2}\right)F \tag{2}$$

where $\Lambda = \partial^2/\partial r^2 + (1/r)\partial/\partial r + (1/r^2)\partial^2/\partial\theta^2$ is the two-dimensional Laplacian, $a_0 = c_{11}/(c_{13} + c_{44})$, $b_0 = c_{44}/(c_{13} + c_{44})$, ρ is the density, and the two displacement functions ψ and F satisfy, respectively,

$$\left(c_{66}\Lambda + c_{44}\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2}\right)\psi = 0 \tag{3}$$

$$\left\{c_{11}c_{44}\Lambda^2 + [c_{11}c_{33} + c_{44}^2 - (c_{13} + c_{44})^2]\Lambda\frac{\partial^2}{\partial z^2} + c_{33}c_{44}\frac{\partial^4}{\partial z^4} - \rho\left[(c_{11} + c_{44})\Lambda + (c_{13} + c_{44})\frac{\partial^2}{\partial z^2}\right]\frac{\partial^2}{\partial t^2} + \rho^2\frac{\partial^4}{\partial t^4}\right\}F = 0 \tag{4}$$

It can be assumed that

$$\psi = h^2\psi_n(\zeta)J_n(\kappa\xi)\sin(n\theta)\exp(i\omega t)$$
$$F = h^3F_n(\zeta)J_n(\kappa\xi)\cos(n\theta)\exp(i\omega t) \tag{5}$$

where $\xi = r/a$ and $\zeta = z/h$ are the dimensionless coordinates, $J_n(x)$ is Bessel function of the first kind, n is an integer, $\psi_n(\zeta)$ and $F_n(\zeta)$ are unknown functions, ω is the circular frequency, and κ is to be determined from the boundary conditions at $r = a$. Strictly speaking, Eq. (5) should be a summation over n from 0 to ∞; however, one typical term is enough because of the orthogonality of trigonometric functions. In contrast to Ding and Xu, [6] we do not set $\psi = 0$ here to include the in-plane vibration simultaneously. It is also noted that the solution form assumed in Eq. (5) is a natural result of the method of separation of variables, which insures the completion of the solution.

Substituting Eq. (5) into Eqs. (3) and (4) yields

$$\psi_n''(\zeta) - [(c_{66}/c_{44})k - \Omega^2]\psi_n(\zeta) = 0 \tag{6}$$

$$F_n^{(4)}(\zeta) - g_1F_n''(\zeta) + g_2F_n(\zeta) = 0 \tag{7}$$

where $\Omega = \omega h\sqrt{\rho/c_{44}}$ is the nondimensional frequency, $k = t_0^2\kappa^2$, $t_0 = h/a$ is the thickness-to-radius ratio, a prime denotes differentiation with respect to ζ, and

$$g_1 = \frac{(c_{11}c_{33} - 2c_{13}c_{44} - c_{13}^2)k}{c_{33}c_{44}} - \frac{\Omega^2(c_{33}+c_{44})}{c_{33}}$$
$$g_2 = \frac{c_{11}k^2 - (c_{11}+c_{44})k\Omega^2 + \Omega^4 c_{44}}{c_{33}} \tag{8}$$

The solutions of Eqs. (6) and (7) are

$$\psi_n(\zeta) = A_3\cosh(\lambda_3\zeta) + B_3\sinh(\lambda_3\zeta) \tag{9}$$

$$F_n(\zeta) = A_1\cosh(\lambda_1\zeta) + A_2\cosh(\lambda_2\zeta) + B_1\sinh(\lambda_1\zeta) + B_2\sinh(\lambda_2\zeta) \tag{10}$$

where the eigenvalues $\lambda_i (i = 1, 2, 3)$ are determined by

$$\lambda_{1,2} = \sqrt{\left(g_1\mu\sqrt{g_1^2 - 4g_2}\right)/2}, \qquad \lambda_3 = \sqrt{(c_{66}/c_{44})k - \Omega^2} \tag{11}$$

It is noted here that Ding and Xu [6] employed exponential functions to express the solutions. Here, however, we use hyperbolic sine and cosine functions, which will benefit the following analysis as will be shown later.

The expressions for displacements can be obtained from Eqs. (2) and (5)

$$u_r = -ht_0\left[\frac{n}{\xi}\psi_n(\zeta)J_n(\kappa\xi) + F_n'(\zeta)\bar{J}_n(\kappa\xi)\right]\cos(n\theta)\exp(i\omega t)$$
$$u_\theta = ht_0\left[\psi_n(\zeta)\bar{J}_n(\kappa\xi) + \frac{n}{\xi}F_n'(\zeta)J_n(\kappa\xi)\right]\sin(n\theta)\exp(i\omega t)$$
$$w = h[b_0F_n''(\zeta) - (a_0k - b_0\Omega^2)F_n(\zeta)]J_n(\kappa\xi)\cos(n\theta)\exp(i\omega t) \tag{12}$$

where a bar over the function $J_n(\kappa\xi)$ denotes differentiation with respect to ξ. The expressions for stresses can be further obtained by virtue of the constitutive relations, but these are omitted here to save space.

Then, for the following free surface conditions

$$\sigma_z = \tau_{zr} = \tau_{\theta z} = 0, \qquad \zeta = -1/2, 1/2 \tag{13}$$

it is obtained that

$$b_0F_n'''(-1/2) + (b_0\Omega^2 - a_0k + c_{13}k/c_{33})F_n'(-1/2) = 0, \qquad t_0\psi_n'(-1/2)\bar{J}_n - \frac{nt_0}{\xi}[(b_0-1)F_n''(-1/2) + (b_0\Omega^2 - a_0k)F_n(-1/2)]J_n = 0$$
$$-\frac{nt_0}{\xi}\psi_n'(-1/2)J_n + t_0[(b_0-1)F_n''(-1/2) + (b_0\Omega^2 - a_0k)F_n(-1/2)]\bar{J}_n = 0, \qquad b_0F_n'''(1/2) + (b_0\Omega^2 - a_0k + c_{13}k/c_{33})F_n'(1/2) = 0$$
$$t_0\psi_n'(1/2)\bar{J}_n - \frac{nt_0}{\xi}[(b_0-1)F_n''(1/2) + (b_0\Omega^2 - a_0k)F_n(1/2)]J_n = 0$$
$$-\frac{nt_0}{\xi}\psi_n'(1/2)J_n + t_0[(b_0-1)F_n''(1/2) + (b_0\Omega^2 - a_0k)F_n(1/2)]\bar{J}_n = 0 \tag{14}$$

which yield

$$\psi_n'(\zeta_i) = A_3\lambda_3\sinh(\lambda_3\zeta_i) + B_3\lambda_3\cosh(\lambda_3\zeta_i) = 0 \tag{15}$$

$$b_0F_n'''(\zeta_i) + [b_0\Omega^2 - (a_0 - c_{13}/c_{33})k]F_n'(\zeta_i) = G_1(\lambda_1)[A_1\sinh(\lambda_1\zeta_i) + B_1\cosh(\lambda_1\zeta_i)] + G_1(\lambda_2)[A_2\sinh(\lambda_2\zeta_i) + B_2\cosh(\lambda_2\zeta_i)] = 0$$
$$(b_0-1)F_n''(\zeta_i) + (b_0\Omega^2 - a_0k)F_n(\zeta_i) = G_2(\lambda_1)[A_1\cosh(\lambda_1\zeta_i) + B_1\sinh(\lambda_1\zeta_i)] + G_2(\lambda_2)[A_2\cosh(\lambda_2\zeta_i) + B_2\sinh(\lambda_2\zeta_i)] = 0 \tag{16}$$

where $i = 1, 2$, $\zeta_1 = -1/2$, $\zeta_2 = 1/2$, and

$$G_1(\lambda) = b_0\lambda^3 + [b_0\Omega^2 + (c_{13}/c_{33} - a_0)k]\lambda$$
$$G_2(\lambda) = (b_0 - 1)\lambda^2 + b_0\Omega^2 - a_0k \tag{17}$$

To make sure that Eqs. (15) and (16) have nontrivial solutions, the corresponding coefficient determinants should vanish, which yields the following frequency equations:

$$\lambda_3^2\sinh(\lambda_3) = 0 \tag{18}$$

$$[G_{12}\sinh(\lambda_1/2)\cosh(\lambda_2/2) - G_{21}\sinh(\lambda_2/2)\times\cosh(\lambda_1/2)][G_{12}\cosh(\lambda_1/2)\sinh(\lambda_2/2) - G_{21}\cosh(\lambda_2/2)\times\sinh(\lambda_1/2)] = 0 \tag{19}$$

where $G_{12} = G_1(\lambda_1)G_2(\lambda_2)$ and $G_{21} = G_1(\lambda_2)G_2(\lambda_1)$. The different product factors in Eqs. (18) and (19) may correspond to different vibration modes, as will be discussed immediately.

It is clear that $\lambda_3 = 0$ is also a root of the factor $\sinh(\lambda_3) = 0$, thus Eq. (17) is equivalent to $\sinh(\lambda_3) = 0$, from which we obtain

$$\Omega = \sqrt{(c_{66}/c_{44})k + p^2\pi^2} \qquad (p = 0, 1, 2, \Lambda) \tag{20}$$

and $A_1 = B_1 = A_2 = B_2 = 0$. Equation (19) determines the natural frequencies of the in-plane vibration of a circular plate with particular boundary conditions at $r = a$ to be determined in the following.

From Eqs. (12) and the expressions for stresses, we find that if κ satisfies

$$J_n(\kappa) = 0 \tag{21}$$

then on the boundary $r = a(\xi = 1)$, boundary condition (ii), as shown in Eq. (1b), is satisfied. On the other hand, if κ satisfies

$$\bar{J}_n(\kappa) = 0 \tag{22}$$

then on the boundary $r = a(\xi = 1)$, boundary condition (i), as shown in Eq. (1a), is satisfied. Note that boundary condition (i) is very similar to the usual elastic simple support.

The corresponding vibration mode can be obtained as

$$u_r = -ht_0\frac{n}{\xi}\psi_n(\zeta)J_n(\kappa\xi)\cos(n\theta)\exp(i\omega t)$$
$$u_\theta = ht_0\psi_n(\zeta)\bar{J}_n(\kappa\xi)\sin(n\theta)\exp(i\omega t), \qquad w = 0 \tag{23}$$

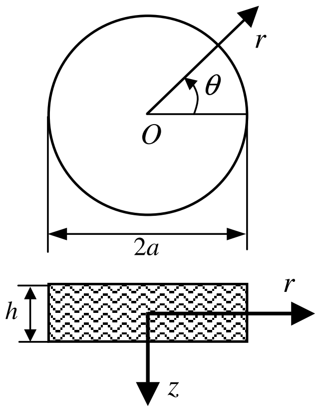

Fig. 1 The geometry and coordinates of a circular plate.

where $\psi_n = A_3 \cosh(\lambda_3 \zeta)$ when p is even and $\psi_n = B_3 \sinh(\lambda_3 \zeta)$ when p is odd. It is seen that the corresponding volume strain

$$e = \frac{\partial u_r}{\partial r} + \frac{(1/r)\partial u_\theta}{\partial \theta} + \frac{u_r}{r} + \frac{\partial w}{\partial z} = 0$$

This type of mode is also known as the thickness-shear motion [10].

According to Mindlin's plate theory, the frequency equation of the in-plane vibration can be obtained as follows:

$$\Omega = \sqrt{(c_{66}/c_{44})k + 12k_t^2} \tag{24}$$

where k_t^2 is the shear correction factor. It can be seen that if $k_t^2 = \pi^2/12$ and $p = 1$, then Eq. (24) is identical to Eq. (20). The value of the shear factor, $k_t = \pi/\sqrt{12}$, is just what was used by Mindlin and Deresiewcz [10].

For the frequency equation resulted from the following factor, see Eq. (19)

$$G_{12} \sinh(\lambda_1/2) \cosh(\lambda_2/2) - G_{21} \sinh(\lambda_2/2) \cosh(\lambda_1/2) = 0 \tag{25}$$

and $A_3 = B_3 = B_1 = B_2 = 0$, it is obtained that

$$\psi_n(\zeta) = 0, \qquad F_n(\zeta) = A_1 \cosh(\lambda_1 \zeta) + A_2 \cosh(\lambda_2 \zeta) \tag{26}$$

Substituting the previous equation into Eq. (12), we find

$$u_r(\zeta) = -u_r(-\zeta), \qquad u_\theta(\zeta) = -u_\theta(-\zeta), \qquad w(\zeta) = w(-\zeta) \tag{27}$$

which implies the corresponding vibration mode is antisymmetric with respect to the middle plane $\zeta = 0$. Thus Eq. (25) determines the natural frequencies of the antisymmetric out-of-plane vibration of a circular plate. Note that the use of hyperbolic sine and cosine functions allows one to obtain the parity of the solution easily.

The particular boundary conditions are the same as those discussed by Ding and Xu [6] for the general out-of-plane vibration, and are contrary to the in-plane vibration, i.e., when κ satisfies Eq. (21), boundary condition (i) holds, and when κ satisfies Eq. (22) boundary condition (ii) holds.

For the frequency equation resulted from the following factor, see Eq. (19)

$$G_{21} \sinh(\lambda_1/2) \cosh(\lambda_2/2) - G_{12} \sinh(\lambda_2/2) \cosh(\lambda_1/2) = 0 \tag{28}$$

and $A_3 = B_3 = A_1 = A_2 = 0$, it is shown that

$$\psi_n(\zeta) = 0, \qquad F_n(\zeta) = B_1 \sinh(\lambda_1 \zeta) + B_2 \sinh(\lambda_2 \zeta) \tag{29}$$

In this case, it can be found that

$$u_r(\zeta) = u_r(-\zeta), \qquad u_\theta(\zeta) = u_\theta(-\zeta), \qquad w(\zeta) = -w(-\zeta) \tag{30}$$

which implies the mode is symmetric with respect to the middle plane $\zeta = 0$. Thus Eq. (28) determines the natural frequencies of the symmetric out-of-plane vibration of a circular plate. The particular boundary conditions are the same as that of the antisymmetric out-of-plane vibration. The symmetric mode frequencies determined from Eq. (28) usually cannot be predicted by plate theories due to their basic assumptions.

Table 1 The fundamental frequencies Ω corresponding to boundary condition (i)

t_0	In-plane			Antisymmetric			Symmetric		
	$n=1$	$n=2$	$n=3$	$n=1$	$n=2$	$n=3$	$n=1$	$n=2$	$n=3$
0.1	0.2013	0.3339	0.4593	0.0776	0.1364	0.2052	0.7205	0.9637	1.1941
0.2	0.4026	0.6678	0.9186	0.2874	0.4823	0.6937	1.4294	1.8980	2.3289
0.3	0.6039	1.0018	1.3780	0.5837	0.9362	1.2970	2.1123	2.7621	3.3118
0.4	0.8052	1.3357	1.8373	0.9286	1.4359	1.9369	2.7494	3.4950	4.0293
0.5	1.0065	1.6695	2.2966	1.2988	1.9529	2.5853	3.3143	4.0433	4.5000
0.6	1.2078	2.0035	2.7559	1.6812	2.4748	3.2325	3.7803	4.4307	4.8733
0.7	1.4091	2.3375	3.2152	2.0690	2.9964	3.8757	4.1409	4.7402	5.2464
0.8	1.6104	2.6714	3.6745	2.4585	3.5158	4.5145	4.4202	5.0345	5.6507
0.9	1.8117	3.0053	4.1339	2.8479	4.0323	5.1490	4.6555	5.3419	6.0908
1.0	2.0130	3.0092	4.5932	3.2362	4.5459	5.7798	4.8754	5.6716	6.5627

Table 2 The fundamental frequencies Ω corresponding to boundary condition (ii)

t_0	In-plane			Antisymmetric			Symmetric		
	$n=1$	$n=2$	$n=3$	$n=1$	$n=2$	$n=3$	$n=1$	$n=2$	$n=3$
0.1	0.4189	0.5615	0.6975	0.0183	0.0498	0.0927	0.3469	0.5749	0.7896
0.2	0.8378	1.1230	1.3951	0.0718	0.1892	0.3392	0.6926	1.1440	1.5636
0.3	1.2568	1.6844	2.0926	0.1566	0.3955	0.6798	1.0357	1.7006	2.3023
0.4	1.6757	2.2459	2.7902	0.2673	0.6459	1.0693	1.3749	2.2365	2.9775
0.5	2.0946	2.8074	3.4877	0.3988	0.9234	1.4821	1.7084	2.7406	3.5525
0.6	2.5135	3.3689	4.1853	0.5460	1.2169	1.9052	2.0343	3.1991	4.0008
0.7	2.9325	3.9304	4.8828	0.7050	1.5195	2.3318	2.3505	3.5976	4.3360
0.8	3.3514	4.4918	5.5804	0.8728	1.8269	2.7589	2.6541	3.9279	4.6039
0.9	3.7703	5.0533	6.2779	1.0468	2.1366	3.1849	2.9421	4.1942	4.8465
1.0	4.1892	5.6148	6.9755	1.2254	2.4472	3.6092	3.2113	4.4129	5.0886

Numerical Investigation

As in Ding and Xu, [6] the free vibration of a transversely isotropic circular plate with elastic constants $c_{11} = 139$ GPa, $c_{12} = 77.8$ GPa, $c_{13} = 74.3$ GPa, $c_{33} = 115$ GPa, and $c_{44} = 25.6$ GPa is considered here for the purpose of numerical calculation. It is noted here that because the present analysis is three-dimensional, we can calculate an infinite number of frequencies from the three frequency equations, i.e., Eq. (20) for the in-plane vibration, Eq. (25) for the antisymmetric out-of-plane vibration, and Eq. (28) for the symmetric out-of-plane vibration. Tables 1 and 2 just list the lowest natural frequencies (fundamental frequencies) of each vibration mode for several combinations of parameters.

It is shown that the fundamental frequencies of the in-plane vibration (thickness-shear motion) for boundary condition (i) are always smaller than those for boundary condition (ii), which, however, is contrary to the case of the out-of-plane vibration (antisymmetric or symmetric motion). This follows that boundary condition (i) imposes a constraint on the transverse displacement, yielding a relative larger rigidity for the out-of-plane motion, wheras boundary condition (ii) imposes a constraint on the radial displacement, which leads to a relative larger stiffness for the in-plane vibration.

Xu [15] presented the FEM results of this transversely isotropic circular plate for $n = 1$ using the commercial software ANSYS. His results agree well with those presented in Table 1. Thus, the theoretical deduction in this paper can be verified.

Conclusions

In this note, we reconsider the problem of free vibration of a transversely isotropic circular plate. We find that the out-of-plane vibration can be further divided into two groups, i.e., the antisymmetric mode vibration and the symmetric mode vibration. The separation of three different catalogs of vibration (in-plane, antisymmetric, and symmetric out-of-plane) is very important because the design of practical apparatuses may require particular capability or performance.

It is noted here that when n is taken as a noninteger, the present method can be used to analyze the free vibration of sectorial plates with particular boundary conditions assumed on the straight boundaries.

Because no assumption on the deformation of plate is introduced a priori, the present analysis can be a benchmark to clarify any two-dimensional approximate theories or numerical methods.

Acknowledgment

The work was supported by the National Natural Science Foundation of China (Nos. 10432030 and 50475104).

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